Math 121 **Test #3 Answers** **Fall 2008**

$(34, 55, 73, 79.25, 93)$, $\overline{x} = 69.4$.

1. (15 pts) $(3, 9, 12, 13, 15)$, $\overline{x} = 10.9$.

   This problem concerns means and hypothesis testing, and it involves only one sample. The sample size is greater than 30, so you may use z or t. You may do the calculations using `T-Test`. The seven steps are

   Step 1. $\mu =$ the average weight loss under the new plan.
   $H_0 : \mu = 4.2$.
   $H_1 : \mu > 4.2$.

   Step 2. $\alpha = 0.05$.

   Step 3. The best choice of test statistic is

   $$z = \frac{\overline{x} - \mu_0}{s/\sqrt{n}},$$

   although I would accept t in place of z even though nothing was said about the population being normal.

   Step 4. Calculate

   $$\begin{aligned} z &= \frac{4.8 - 4.2}{1.5/\sqrt{36}} \\ &= \frac{0.6}{0.25} \\ &= 2.4. \end{aligned}$$

   Step 5. p-value $=$ `normalcdf(2.4,E99)` $= 0.0082$. If you used t in step 3, then here you should calculate `tcdf(2.4,E99,35)` $= 0.0109$.

   Step 6. Reject H_0.

   Step 7. The new weight-loss plan produces an average loss of more than 4.2 lbs in the first month.

2. (8 pts) $(0, 3.5, 8, 8, 8)$, $\overline{x} = 6.1$.

   You may use the function `ZInterval`. I would also accept `TInterval`.

   The formula is

   $$\overline{x} \pm z_{0.025}\frac{s}{\sqrt{n}}.$$

   Calculate

   $$\begin{aligned} \overline{x} \pm z_{0.025}\frac{s}{\sqrt{n}} &= 4.2 \pm 1.960\frac{1.5}{\sqrt{36}} \\ &= 4.2 \pm 0.49. \end{aligned}$$

3. (15 pts) $(5, 9.25, 10.5, 13.75, 15)$, $\overline{x} = 11.3$.

   This problem concerns proportions and hypothesis testing, and it involves only one sample. You may do the calculations using `1-PropZTest`. The seven steps are

   Step 1. $p =$ the proportion of Virginia drivers who wear their seat belts.
   $H_0 : p = 0.81$.
   $H_1 : p < 0.81$.

   Step 2. $\alpha = 0.05$.

   Step 3. The only choice of test statistic is

   $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

   Step 4. The sample proportion is $\hat{p} = \frac{410}{521} = 0.7869$. Calculate

   $$\begin{aligned} z &= \frac{0.7869 - 0.81}{\sqrt{\frac{(0.81)(0.19)}{521}}} \\ &= \frac{-0.0231}{0.0172} \\ &= -1.344. \end{aligned}$$

   Step 5. p-value = `normalcdf(-E99,-1.344)` $= 0.0895$.

   Step 6. Accept H_0.

   Step 7. The seat-belt usage rate in Virginia is 81%.

4. (12 pts) $(0, 3, 9.5, 12, 12)$, $\overline{x} = 7.8$.

   (a) (8 pts) You may use the function `1-PropZInt`.
   The formula is
   $$\hat{p} \pm z_{0.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$
   Calculate
   $$\begin{aligned} \hat{p} \pm z_{0.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.7869 \pm 1.645\sqrt{\frac{(0.7869)(0.2131)}{521}} \\ &= 0.7869 \pm 0.0295. \end{aligned}$$

   (b) (4 pts) The margin of error is 0.0295. If you used `1-PropZInt`, then your confidence interval was $(.75744, .81646)$. In that case, calculate $(0.81646 - 0.75744)/2 = 0.2951$.

5. (8 pts) $(0, 6, 8, 8, 8)$, $\overline{x} = 6.5$.

   (a) With $df = 6$, $P(t > 2.5) =$ `tcdf(2.5,E99,6)` $= 0.0233$.

   (b) With $df = 10$, $P(-1 < t < 1) =$ `tcdf(-1,1,10)` $= 0.6591$.

6. (6 pts) $(0, 1, 3.5, 5.75, 6)$, $\overline{x} = 3.2$.

   (a) It would be better to use independent samples. There is no reasonable way to pair the men in the one sample with the women in the other sample.

   (b) It would be better to use paired samples. It is natural to get the math SAT score and the verbal SAT score from the same students and compute the differences between the two scores. The alternative would be to collect one sample of students and get their math SAT scores and then collect another sample of students and get their verbal SAT scores. There are good reasons not to do it that way.

7. (15 pts) $(0, 8.25, 10.5, 12, 15)$, $\overline{x} = 10.0$.

   This problem concerns proportions (the non-violent crime rate) and hypothesis testing. It also involves two samples (before and after the new law). You may use `2-PropZTest` to do the calculations. The seven steps are

   Step 1. $p_1 =$ the non-violent crime rate before the new law.
   $p_2 =$ the non-violent crime rate after the new law.
   $H_0 : p_1 = p_2$.
   $H_1 : p_1 < p_2$. (Rate was lower before the new law.)

   Step 2. $\alpha = 0.01$.

   Step 3. The test statistic is

   $$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n} + \frac{1}{n_2}\right)}},$$

   where $\hat{p}$ is the pooled estimate for the common value of p_1 and p_2.

   Step 4. The values of $\hat{p}_1$ and $\hat{p}_2$ are $\frac{52}{168} = 0.3095$ and $\frac{55}{145} = 0.3793$, respectively. Next, compute $\hat{p} = \frac{52+55}{168+145} = \frac{107}{313} = 0.3419$. Then calculate

   $$\begin{aligned} z &= \frac{0.3095 - 0.3793}{\sqrt{(0.3419)(0.6581)\left(\frac{1}{168} + \frac{1}{145}\right)}} \\ &= \frac{-0.0698}{0.0538} \\ &= -1.298. \end{aligned}$$

   Step 5. p-value $=$ `normalcdf(-E99,-1.298)` $= 0.0971$.

Step 6. Accept H_0.

Step 7. The non-violent crime rate did not increase after the new law.

8. (21 pts) $(4, 11, 14.5, 16, 21)$, $\overline{x} = 13.5$.

Before doing anything else, it would be good to enter the data into the calculator. Enter the first sample into list $\mathtt{L_1}$ and the second sample into $\mathtt{L_2}$. You could then use `1-Var Stats` for each list and get $\overline{x}_1 = 58.9$, $s_1 = 9.927$, $\overline{x}_2 = 58.6$, and $s_2 = 12.195$.

(a) (3 pts) To get the QQ plot, use `STAT PLOT` and select the last graph icon. When the list is $\mathtt{L_1}$, the graph is pretty straight. When the list is $\mathtt{L_2}$, the graph is almost as straight, with a small crooked part in the upper right area. In both cases, they indicate that the underlying populations are likely to be normal.

(b) (15 pts) This problem involves means and hypothesis testing. There are two samples. The samples are small and the QQ plots indicated that the normality assumption is reasonable. Therefore, we should use the t distribution. On the calculator you may use the function `2-SampTTest`. The seven steps are

Step 1. $\mu_1 =$ the average high temperature 1988 - 1997.
$\mu_2 =$ the average high temperature 1998 - 2007.
$H_0 : \mu_1 = \mu_2$.
$H_1 : \mu_1 \neq \mu_2$. (Has the average temperature "changed?")

Step 2. $\alpha = 0.01$.

Step 3. The test statistic is

$$t = \frac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}.$$

If you used s_1^2 and s_2^2 instead of s_p, that was ok.

Step 4. First, find

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = 11.119.$$

Then calculate

$$\begin{aligned} z &= \frac{58.9 - 58.6}{11.119\sqrt{\frac{1}{10} + \frac{1}{10}}} \\ &= \frac{0.3}{4.973} \\ &= 0.0603. \end{aligned}$$

Step 5. p-value $= 2 \times$ `tcdf(0.0603,E99,18)` $= 2 \times 0.4763 = 0.9526$.

Step 6. Accept H_0.

Step 7. The average high temperature for Nov. 21 has not changed from the decade 1988 - 1997 to the decade 1998 - 2007.

(c) (3 pts) The two sample standard deviations are $s_1 = 9.927$ and $s_2 = 12.195$ and the pooled estimate of σ is $s_p = 11.119$. If you used `2-SampTTest`, then these values were all in the display.